

<p>Marks : 40</p>	<p><b>SYJC FEB' 19</b>  <b>Subject : Maths – II</b>  <b>Regression, Random Variable</b></p>	<p>Duration : 1.5 Hours.                   Set – A SOLUTION</p>
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**Q.1. Solve any Three : (2 Marks each) (06)**

1. Given :  $\bar{x} = 53, \bar{y} = 28, b_{YX} = -1.5$  and  $b_{XY} = -0.2$

**Correlations coefficient between X and Y :**

$$\begin{aligned}
 r &= \pm \sqrt{b_{yx} \cdot b_{xy}} \\
 &= \pm \sqrt{(-1.5)(-0.2)} \\
 &= \pm \sqrt{0.3} \\
 \therefore r &= -0.5477 \quad \dots (\because b_{yx} \text{ and } b_{xy} \text{ are negative})
 \end{aligned}$$

2. Given :  $\bar{x} = 199, \bar{y} = 94, \sum(x_i - \bar{x})^2 = 1298, \sum(y_i - \bar{y})^2 = 600, \sum(x_i - \bar{x})(y_i - \bar{y}) = -262$

**The line of regression of Y on X :**

$$\begin{aligned}
 b_{yx} &= \frac{\sum(x - \bar{x})\sum(y - \bar{y})}{\sum(x_i - \bar{x})^2} \\
 &= \frac{-262}{1298} \\
 &= -0.2018
 \end{aligned}$$

Now,  $y - \bar{y} = b_{yx} (x - \bar{x})$

$$\begin{aligned}
 \therefore y - 92 &= 0.2018 (x - 199) \\
 \therefore y &= -0.2018x + 40.1582 + 94 \\
 \therefore y &= -0.2018x + 134.1582 \\
 \therefore y &= 134.1582 - 0.2018x
 \end{aligned}$$

3. Given :  $\bar{x} = 10, \bar{y} = 90, \sigma_x = 3, \sigma_y = 12, r = 0.8$

**(i) Regression equation of Y on X :**

$$\begin{aligned}
 b_{yx} &= r \cdot \frac{\sigma_y}{\sigma_x} \\
 \therefore b_{yx} &= 0.8 \times \frac{12}{3} = 3.2
 \end{aligned}$$

Now,  $y - \bar{y} = b_{yx} (x - \bar{x})$

$$\begin{aligned}
 \therefore y - 90 &= 3.2 (x - 10) \\
 \therefore y &= 3.2x - 32 + 90 \\
 \therefore y &= 3.2x + 58 \\
 \therefore y &= 58 + 3.2x
 \end{aligned}$$

**(ii) Regression equation of X on Y :**

$$\begin{aligned}
 b_{xy} &= r \cdot \frac{\sigma_x}{\sigma_y} \\
 \therefore b_{xy} &= 0.8 \times \frac{3}{12} = 0.2
 \end{aligned}$$

Now,  $x - \bar{x} = b_{xy} (y - \bar{y})$

$$\begin{aligned}
 \therefore x - 10 &= 0.2 (y - 90) \\
 \therefore x &= 0.2y - 18 + 10
 \end{aligned}$$

$$\therefore x = 0.2y - 8$$

$$\therefore x = -8 + 0.2y$$

4.  $b_{yx} + b_{xy} = 1.30, r = 0.75$

$$\frac{b_{yx} + b_{xy}}{2} = \frac{1.30}{2}, r = 0.75$$

$$\therefore \frac{b_{yx} + b_{xy}}{2} < r$$

Hence, the data is inconsistent.

5. We know that the co-ordinates of point of intersection of the two lines are  $\bar{x}$  and  $\bar{y}$ , the means of X and Y.

The regression equation are

$$3x + 2y - 26 = 0$$

$$6x + y - 31 = 0 \quad \text{and}$$

Solving these equations simultaneously, we get

$$6x + 4y - 52 = 0$$

$$6x + y - 31 = 0$$

$$\begin{array}{r} - \quad - \quad + \\ \hline 3y - 21 = 0 \end{array}$$

$$\therefore 3x = 21$$

$$\text{i.e. } y = 7$$

and  $x = 4$

hence, the means of X and Y are  $\bar{x} = 4$  and  $\bar{y} = 7$ .

**Q.2. Solve any Four : (3 Marks each)**

**(12)**

1. Here, the regression lines are specified. So,  $b_{YX} = \frac{4}{3}$  and  $b_{XY} = \frac{1}{3}$

$$\therefore r^2 = b_{YX} \cdot b_{XY} = \frac{4}{3} \cdot \frac{1}{3} = \frac{4}{9}$$

$$\therefore r = +\frac{2}{3} \quad \dots (\because b_{yx} \text{ and } b_{xy} \text{ are positive})$$

You know that

$$b_{YX} = r \cdot \frac{\sigma_Y}{\sigma_X}$$

$$\therefore \frac{4}{3} = \frac{2}{3} \cdot \frac{\sigma_Y}{2}$$

$$\therefore \sigma_Y = 4$$

$$\therefore \sigma_Y^2 = 16$$

2. Here, we need to find line of regression of Y on X, which is given as  $Y = a + b_{YX} X$ .

$$\text{Where } b_{YX} = \frac{\text{cov}(X, Y)}{\sigma_X^2}$$

$$= \frac{\sum (x_i - \bar{x})(x_i - \bar{y})}{n \sigma_X^2}$$

$$= \frac{1220}{130}$$

$$= 0.9384$$

and a

$$= \bar{y} - b_{yx} \bar{x}$$

$$= 142 - (0.9384) 53$$

$$= 92.2615$$

Therefore, regression equation of Y on X is

$$Y = 92.2615 + 0.9384X$$

Now, the estimate of blood pressure of woman with age 47 years is

$$Y = 92.2615 + 0.9384 \times 47$$

$$= 136.3692$$

3. Given :  $n = 8, \bar{x} = 20, \bar{y} = 36, \sum (x_i - \bar{x})(y_i - \bar{y}) = 120, \sigma_x = 2, \sigma_y = 3$

$$b_{yx} = \frac{\text{Cov}(x, y)}{\sigma_x^2}$$

$$\text{Now, Cov}(x, y) = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n}$$

$$= \frac{120}{8} = 15$$

$$\therefore b_{yx} = \frac{15}{(2)^2} = \frac{15}{4}$$

$$= 3.75$$

**Line of regression of Y on X :**

$$y - \bar{y} = b_{yx} (x - \bar{x})$$

$$\therefore y - 36 = 3.75 (x - 20)$$

$$\therefore y = 3.75x - 75 + 36$$

$$\therefore y = 3.75x - 39$$

4.  $\bar{x} = 53, \bar{y} = 28, b_{yx} = -1.5, b_{xy} = -0.2$

Using regression equation of Y on X, we estimate Y when X = 50.

$$\text{Now, } y - \bar{y} = b_{yx} (x - \bar{x})$$

$$\therefore y - 28 = -1.5(x - 53)$$

$$\therefore y = -1.5x + 79.5 + 28$$

$$\therefore y = -1.5x + 107.5$$

**Estimate of Y when X = 50 :**

$$\text{Put } x = 50 \text{ in } y = -1.5x + 107.5$$

$$\therefore y = -1.5 \times 50 + 107.5$$

$$\therefore y = -75 + 107.5$$

$$\therefore y = 32.5$$

5. Given :  $b_{yx} = -0.75, b_{xy} = -1.1, r = ?$

**Correlation coefficient :**

$$r = \pm \sqrt{b_{yx} \cdot b_{xy}}$$

$$\therefore r = \pm \sqrt{(-0.75) \times (-1.1)}$$

$$\therefore r = \pm \sqrt{0.825}$$

$$\therefore r = -0.9082 (\because b_{yx} \text{ and } b_{xy} \text{ are negative})$$

**Q.3. Solve any One : (4 Marks each)**

**(04)**

1. Given :  $n = 7, \sum x_i = 105, \sum y_i = 409, \sum x_i^2 = 1681, \sum y_i^2 = 39350, \sum x_i y_i = 8075$

$$\bar{x} = \frac{\sum x_i}{n} = \frac{105}{7} = 15, \bar{y} = \frac{\sum y_i}{n} = \frac{409}{7} = 58.428$$

**Regression coefficient of Y on X :**

$$b_{yx} = \frac{\frac{\sum x_i y_i}{n} - (\bar{x})(\bar{y})}{\frac{\sum x_i^2}{n} - (\bar{x})^2}$$

$$= \frac{\frac{8075}{7} - 15 \times 58.428}{\frac{1681}{7} - (15)^2}$$

$$= \frac{1153.571 - 876.42}{240.143 - 225}$$

$$= \frac{277.151}{15.143} = 18.3023$$

**Regression equation of Y on X :**

$$y - \bar{y} = b_{yx} (x - \bar{x})$$

$$\therefore y - 58.428 = 18.3023 (x - 15)$$

$$\therefore y = 18.3023x - 274.5345 + 58.428$$

$$\therefore y = 18.3023x - 216.1065$$

$$\therefore y = -216.1065 + 18.3023x.$$

**2.** Given :  $10x - 4y = 80$ ,  $10y - 9x = -40$

**(i)**  $\bar{x}$  and  $\bar{y}$  :

$$10x - 4y = 80 \quad \dots (1)$$

$$-9x + 10y = -40 \quad \dots (2)$$

Multiplying equation (1) by 9 and equation (2) by 10 and then adding them, we get

$$90x - 36y = 720 \quad \dots (1)$$

$$-90x + 100y = -420 \quad \dots (2)$$

$$\therefore 64y = 320$$

$$\therefore y = \frac{320}{64} = 5$$

Put  $y = 5$  in equation (1), we get

$$10x - 4(5) = 80$$

$$\therefore 10x = 80 + 20$$

$$\therefore x = \frac{100}{10} = 10$$

Hence,  $\bar{x} = 10$ ,  $\bar{y} = 5$

**(ii)**  $b_{yx}$  and  $b_{xy}$

Let regression equation of X on Y be

$$10x - 4y = 80$$

$$\therefore 10x = 4y + 80$$

$$\therefore x = \frac{4}{10}y + 8$$

$$\therefore b_{xy} = 0.4$$

And another equation  $10y - 9x = -40$  be the regression equation of Y on X.

$$\therefore 10y = 9x - 40$$

$$\therefore y = \frac{9}{10}x - 4$$

$$\therefore b_{yx} = \frac{9}{10} = 0.9$$

Hence,  $b_{yx} = 0.9$  and  $b_{xy} = 0.4$ .

**(iii) Coefficient of correlation r :**

$$\begin{aligned} r &= \pm \sqrt{b_{yx} \cdot b_{xy}} \\ &= \pm \sqrt{0.9 \times 0.4} \\ &= \pm \sqrt{0.36} \\ \therefore r &= 0.6 \quad (\because b_{yx} \text{ and } b_{xy} \text{ are positive}) \end{aligned}$$

**(iv) V(X) if V(Y) = 36  $\therefore \sigma_y = 6$  :**

$$\text{Now, } b_{yx} = r \cdot \frac{\sigma_y}{\sigma_x}$$

$$\therefore 0.9 = 0.6 \times \frac{6}{\sigma_x}$$

$$\therefore \frac{0.9}{0.6 \times 6} = \frac{1}{\sigma_x}$$

$$\therefore \frac{1}{4} = \frac{1}{\sigma_x}$$

$$\therefore \sigma_x = 4$$

$$\therefore V(X) = \sigma_x^2 = (4)^2 = 16$$

**3.** Given :  $3x + 2y - 26 = 0$ ,  $6x + y - 31 = 0$

**(i) Means of X and Y :**

$$3x + 2y - 26 = 0 \quad \dots (1)$$

$$6x + y - 31 = 0 \quad \dots (2)$$

Multiplying equation (2) by 2 and subtracting it from the equation (1), we get

$$3x + 2y - 26 = 0 \quad \dots (1)$$

$$12x + 2y - 62 = 0 \quad \dots (2)$$

$$\begin{array}{r} - \quad - \quad + \\ \hline \end{array}$$

$$\therefore -9x + 36 = 0$$

$$\therefore 9x = 36$$

$$\therefore x = \frac{36}{9} = 4$$

Put  $x = 4$  in equation (1),

$$\therefore 3(4) + 2y - 26 = 0$$

$$\therefore 2y + 12 - 26 = 0$$

$$\therefore 2y = 14$$

$$\therefore y = 7$$

Hence,  $\bar{x} = 4$ ,  $\bar{y} = 7$

**(ii) Correlation coefficient between X and Y :**

Let  $3x + 2y - 26 = 0$  be the regression equation of Y on X.

$$\therefore 2y = -3x + 26$$

$$\therefore y = -\frac{3}{2}x + 13$$

$$\therefore b_{yx} = -\frac{3}{2} \quad (\because \text{it is coefficient of } x)$$

and the other equation  $6x + y - 31 = 0$  be the regression equation of X and Y.

$$\therefore 6x = -y + 31$$

$$\therefore x = -\frac{1}{6}y + \frac{31}{6}$$

$$\therefore b_{xy} = -\frac{1}{6} \quad (\because \text{it is coefficient of } y)$$

$$\begin{aligned} \text{Now, } r &= \pm \sqrt{b_{yx} \cdot b_{xy}} \\ &= \pm \sqrt{\left(-\frac{3}{2}\right) \times \left(-\frac{1}{6}\right)} = \pm \sqrt{\frac{1}{4}} \\ &= \pm \frac{1}{2} \end{aligned}$$

$$\therefore r = -0.5 \quad (\because b_{yx} \text{ and } b_{xy} \text{ are negative})$$

**(iii) Estimate of Y for X = 2 :**

Regression line of Y on X is,

$$y - \bar{y} = b_{yx} (x - \bar{x})$$

$$\therefore y - 7 = -\frac{3}{2}(x - 4)$$

$$\therefore y = -\frac{3}{2}x + 6 + 7$$

$$\therefore y = -\frac{3}{2}x + 13$$

Put  $x = 2$  in  $y = -\frac{3}{2}x + 13$

$$\therefore y = -\frac{3}{2} \times 2 + 13$$

$$\therefore y = -3 + 13$$

$$\therefore y = 10$$

**(iv)  $V(Y) = 36 = \sigma_y^2$**

$$\therefore \sigma_y = 6, \text{ Var } (X) = ?$$

Now,  $b_{yx} = r \cdot \frac{\sigma_y}{\sigma_x}$

$$\therefore -\frac{3}{2} = -\frac{1}{2} \times \frac{6}{\sigma_x}$$

$$\therefore 3 = \frac{6}{\sigma_x}$$

$$\therefore \sigma_x = \frac{6}{3} = 2$$

$$\therefore \text{Var } (X) = \sigma_x^2 = 4.$$

**Q.4. Attempt any Two : (2 Marks each)**

**(04)**

- Here, we define r.v. X = number of points appearing on the uppermost face of a fair die. X can take values 1, 2, 3, 4, 5, 6.

Since die is fair, each number has equal probability distribution of X is as shown in the following table.

X = x	1	2	3	4	5	6
P[X = x]	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

2. Define X = number of defective batteries selected by the person.

∴ Range set of X = {0, 1, 2}. The p.m.f. of X is as follows :

$$P(X = 0) = \frac{{}^3C_0 \times {}^5C_2}{{}^8C_2} = \frac{10}{28}$$

$$P(X = 1) = \frac{{}^3C_1 \times {}^5C_1}{{}^8C_2} = \frac{15}{28}$$

$$P(X = 2) = \frac{{}^3C_2 \times {}^5C_0}{{}^8C_2} = \frac{3}{28}$$

Thus, the probability distribution of X is :

X = x	0	1	2
P[X = x]	$\frac{10}{28}$	$\frac{15}{28}$	$\frac{3}{28}$

3. 3 fair coins are tossed simultaneously.

∴ s = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

∴ total outcomes n = 8

X = the number of heads

∴ Range set of X = {0, 1, 2, 3}

$$\therefore P(x = 0) = P\{TTT\} = \frac{1}{8}$$

$$P(x = 1) = P\{HTT, THT, TTH\} = \frac{3}{8}$$

$$P(x = 2) = P\{HHT, HTH, THH\} = \frac{3}{8}$$

$$\therefore P(x = 3) = P\{HHH\} = \frac{1}{8}$$

Hence, the probability distribution of X is as shown in the following table :

X = x	0	1	2	3
P(X = x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

4. Given :  $f(x) = \begin{cases} kx^2(1-x), & 0 < x < 1; \\ 0, & \text{otherwise} \end{cases}$

Here, f(x) is a pdf of r.v. X

$$\therefore \int_0^1 kx^2(1-x) dx = 1$$

$$\therefore \int_0^1 x^2 dx - \int_0^1 x^3 dx = \frac{1}{k}$$

$$\therefore \left[ \frac{x^3}{3} \right]_0^1 - \left[ \frac{x^4}{4} \right]_0^1 = \frac{1}{k}$$

$$\therefore \left( \frac{1}{3} - 0 \right) - \left( \frac{1}{4} - 0 \right) = \frac{1}{k}$$

$$\therefore \frac{1}{3} - \frac{1}{4} = \frac{1}{k}$$

$$\therefore \frac{4-3}{12} = \frac{1}{k}$$

$$\therefore k = 12$$

Hence,  $k = 12$

**Q.5. Attempt any Four : (3 Marks each)**

**(12)**

1. Given pdf of r.v.  $X$  is

$$f(x) = \frac{k}{\sqrt{x}}, 0 < x < 4$$

$$= 0, \text{ elsewhere}$$

We know that,

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

$$\therefore \int_0^4 \frac{k}{\sqrt{x}} dx = 1$$

$$\therefore k \int_0^4 x^{-\frac{1}{2}} dx = 1$$

$$\therefore k \left[ \frac{x^{\frac{1}{2}}}{\frac{1}{2}} \right]_0^4 = 1$$

$$\therefore k \left[ \frac{4^{\frac{1}{2}}}{\frac{1}{2}} - 0 \right] = 1$$

$$\therefore k \times 4 = 1$$

$$\therefore k = \frac{1}{4}$$

2. Let  $X =$  No. of defective bulbs in a sample of 5 bulbs

$$\therefore p = \text{Probability of defective bulb}$$

$$= \frac{5}{25} = \frac{1}{5}$$

$$\therefore q = 1 - p = 1 - \frac{1}{5} = \frac{4}{5}$$

Given :  $n = 5$  (No. of bulbs in a sample)

$$\therefore X \sim B \left( 5, \frac{1}{5} \right)$$

Now,  $p(x) = {}^n C_x p^x q^{n-x}$

$$\therefore p(x) = {}^5 C_x \left( \frac{1}{5} \right)^x \left( \frac{4}{5} \right)^{5-x}$$

(i)  $P$  [Exactly one defective bulb] i.e.  $P[X = 1]$

$$\therefore p(1) = {}^5 C_1 \left( \frac{1}{5} \right)^1 \left( \frac{4}{5} \right)^4$$

$$= 5 \times \frac{1}{5} \times \frac{256}{625}$$

$$= 0.4096$$

Hence, the probability that exactly one bulb is defective in a sample is 0.4096.

(ii)  $P$ [At least one bulb is defective] i.e.  $P[X \geq 1]$

$$P[X \geq 1] = 1 - P[X < 1]$$



$$\begin{aligned}
&= 1 - P(0) \\
&= 1 - p(0) \\
&= 1 - {}^5C_0 \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^5 \\
&= 1 - 1 \times \frac{1024}{3125} \\
&= 1 - 0.32768 \\
&= 0.67232
\end{aligned}$$

Hence, the probability that at least one bulb is defective in a sample of 5 bulbs is 0.67232.

3.  $X \sim P(m = 5), e^{-5} = 0.0067$

Hence,  $p(x) = \frac{e^{-m} m^x}{x!}$

$\therefore p(x) = \frac{e^{-5} 5^x}{x!} = 0.0067 \times \frac{5^x}{x!}$

(i)  $P[X = 5] :$

$$\begin{aligned}
P[X = 5] &= p(5) = 0.0067 \times \frac{(5)^5}{5!} \\
&= 0.0067 \times \frac{3125}{120} \\
&= \frac{20.9375}{120} = 0.1745
\end{aligned}$$

$\therefore P[X = 5] = 0.1745$

(ii)  $P[X \geq 2] :$

$$\begin{aligned}
P[X \geq 2] &= 1 - [P(0) + P(1)] = 1 - [p(0) + p(1)] \\
&= 1 - \left[ 0.0067 \times \frac{5^0}{0!} + 0.0067 \times \frac{5^1}{1!} \right] \\
&= 1 - 0.0067 (1 + 5) \\
&= 1 - 0.0067 \times 6 \\
&= 1 - 0.0402 = 0.9598
\end{aligned}$$

$\therefore P[X \geq 2] = 0.9598$

4. Here,  $X \sim P(m)$

$\therefore p(x) = \frac{e^{-m} m^x}{x!}$

(i) Mean of the distribution :

Given :  $P[X = 1] = P[X = 2]$

$\therefore p(1) = p(2)$

$\therefore \frac{e^{-m} m^1}{1!} = \frac{e^{-m} m^2}{2!}$

$\therefore m = \frac{m^2}{2}$

$\therefore 1 = \frac{m}{2}$

$\therefore m = 2$

Hence, mean of the distribution is 2.

(ii)  $P(X = 0) :$

$$P(x) = \frac{e^{-2}2^x}{x!}, e^{-2} = 0.1353$$

$$\therefore p(x) = 0.1353 \times \frac{2^x}{x!}$$

$$\therefore P[X = 0] = p(0) = 0.1353 \times \frac{2^0}{0!}$$

$$\therefore P[X = 0] = 0.1353$$

5. Let X = No. of days it rains in a week  
p = Probability that it rains

$$= \frac{12}{30} = \frac{2}{5}$$

$$\therefore q = 1 - p = 1 - \frac{2}{5} = \frac{3}{5}$$

Given : n = 7 (No. of days in a week)

$$\therefore X \sim B \left( 7, \frac{2}{5} \right)$$

Hence,  $p(x) = {}^n C_x p^x q^{n-x}$

$$\therefore p(x) = {}^7 C_x \left( \frac{2}{5} \right)^x \left( \frac{3}{5} \right)^{7-x}$$

P [It rains on exactly 3 days of the week] i.e.  $P[X = 3]$  :

$$\begin{aligned} \therefore P[X = 3] &= p(3) = {}^7 C_3 \left( \frac{2}{5} \right)^3 \left( \frac{3}{5} \right)^{7-3} \\ &= \frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times \frac{8}{125} \times \frac{81}{625} \\ &= \frac{35 \times 8 \times 81}{125 \times 625} \\ &= \frac{22680}{78125} = 0.290304 \end{aligned}$$

$$\therefore P[X = 3] = 0.290304$$

Hence, the probability that it rains on exactly 3 days of week is 0.290304.

6. Given,  $X \sim B(n, p)$

$$\therefore p(x) = {}^n C_x p^x q^{n-x}$$

Given : n = 5,  $p(1) = 0.4096$ ,  $p(2) = 0.2048$ , p = ?

Now,  $p(x) = {}^5 C_x p^x q^{n-x}$

$$\therefore p(1) = {}^5 C_1 p^1 q^4$$

$$\therefore p(1) = 5pq^4$$

and  $p(2) = {}^5 C_2 p^2 q^3$

$$\therefore p(2) = 10 \cdot p^2 q^3$$

$$\text{Now, } \frac{P(X=1)}{P(X=2)} = \frac{P(1)}{P(2)} = \frac{5pq^4}{10p^2q^3}$$

$$\therefore \frac{0.4096}{0.2048} = \frac{q}{2p}$$

$$\therefore 2 = \frac{q}{2p}$$

$$\therefore 4p = q$$

$$\therefore 4p = 1 - p$$

$$\therefore 5p = 1$$

$$\therefore p = \frac{1}{5}$$

Hence, the probability of success is  $\frac{1}{5}$ .

**Q.6. Attempt any One : (4 Marks each)**

**(12)**

1. Given that,

$$f(x) = \frac{1}{x^2}, 1 < x < \infty$$

$$= 0, \text{ otherwise.}$$

Cdf of a continuous r.v. X is given by

$$F(x) = \int_{-\infty}^x f(x) dx$$

Now range of X starts at 1

$$\therefore F(x) = \int_1^x f(x) dx$$

$$= \int_1^x \frac{1}{x^2} dx$$

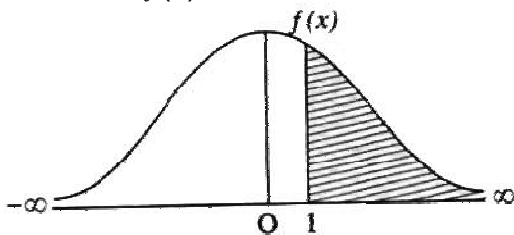
$$= \left[ -\frac{1}{x} \right]_1^x$$

$$= -\frac{1}{x} - (-1)$$

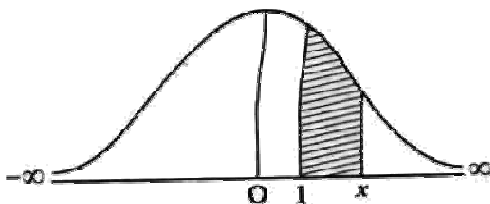
$$= 1 - \frac{1}{x}$$

Hence, cdf of X,  $F(x) = 1 - \frac{1}{x}$

Sketch of  $f(x)$  :



Sketch of  $F(x)$  :



2. Let X = No. of workers suffering from occupational disease.

p = Probability that worker suffering from the disease

$$= 25\% = \frac{25}{100} = \frac{1}{4}$$

$$\therefore q = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}$$

Given : n = 6 (No. Of workmen)

$$\therefore X \sim B \left( 6, \frac{1}{4} \right)$$

$$\text{Now, } p(x) = {}^n C_x p^x q^{n-x}$$

$$\therefore p(x) = {}^6 C_x \left( \frac{1}{4} \right)^x \left( \frac{3}{4} \right)^{6-x}$$

P [Four or more workmen] i.e.  $P[X \geq 4]$  :

$$\begin{aligned} \therefore P[X \geq 4] &= P(4) + P(5) + P(6) \\ &= p(4) + p(5) + p(6) \\ &= {}^6 C_4 \left( \frac{1}{4} \right)^4 \left( \frac{3}{4} \right)^2 + {}^6 C_5 \left( \frac{1}{4} \right)^5 \left( \frac{3}{4} \right)^1 + {}^6 C_6 \left( \frac{1}{4} \right)^6 \left( \frac{3}{4} \right)^0 \\ &= \frac{6 \times 5}{2 \times 1} \times \frac{1}{256} \times \frac{9}{16} + 6 \times \frac{1}{1024} \times \frac{3}{4} + 1 \times \frac{1}{4096} \times 1 \\ &= \frac{15 \times 9}{4096} + \frac{18}{4096} + \frac{1}{4096} \\ &= \frac{135 + 18 + 1}{4096} \\ &= \frac{154}{4096} = 0.0376 \end{aligned}$$

$$\therefore P[X \geq 4] = 0.0376$$

Hence, the probability that 4 or more workmen will contact the disease is 0.0376.

3.  $X$  = No. of accidents

10 accidents take place in 50 days is given.

$$\therefore \text{average no. of accidents per day } m = \frac{10}{50} = 0.2$$

$$\therefore X \sim P(m = 0.2); e^{-0.2} = 0.8187$$

$$\text{Hence, } p(x) = \frac{e^{-m} m^x}{x!}$$

$$\therefore \frac{e^{-0.2} (0.2)^x}{x!} = 0.8187 \times \frac{(0.2)^x}{x!}$$

$$\begin{aligned} P[X \geq 3] &= 1 - P[X = 0] + P(X = 1) + P(X = 2)] \\ &= 1 - [p(0) + p(1) + p(2)] \\ &= 1 - \left[ 0.8187 \times \frac{(0.2)^0}{0!} + 0.8187 \times \frac{(0.2)^1}{1!} + 0.8187 \times \frac{(0.2)^2}{2!} \right] \\ &= 1 - \left[ 0.8187 \left( 1 + 0.2 + \frac{0.04}{2} \right) \right] \\ &= 1 - [0.8187(1.2 + 0.02)] \\ &= 1 - [0.8187 \times 1.22] \\ &= 1 - 0.9988 \\ &= 0.0012 \end{aligned}$$

Hence, the probability that there are three or more accidents per day is 0.0012.

